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ARCTIC SITE SURVEY

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1. RAY TRACING IN THE ARCTIC OCEAN

Ray Tracing Program

In the Arctic Ocean it is known that below the upper layer of several hundred feet the velocity gradient is fairly uniform and moderate with respect to depth and with the exception of extreme low frequency propagation ray tracing affords a very convenient and effective technique for calculating the acoustic intensities.

Aside from a few minor details, a ray tracing program is now completed and is capable of performing the following tasks. Given the locations of a number of sound sources (longitude, latitude, and depth) and the locations of a number of receiver stations (longitude, latitude, and depth), the program evaluates the acoustic intensities at each receiver station corresponding to each of the given sources and the topography of the Arctic Ocean. Input data consists of a typical temperature and salinity profile and the RMS ice roughness over the Arctic Ocean for computing the surface reflection losses.

A typical ray diagram and velocity profile using recent temperature and salinity data at Base Camp (72-0.20N, 148-35.2W) are plotted and included in this report. The units of depth and range are in feet and velocity in feet per second. The source is 1000 feet below the surface and the grazing angles of the rays have one degree increment.

Velocity Profile from Temperature and Salinity Data

While the most widely used formula for computing the velocity from temperature and salinity is the Wilson's ("Equations for the Speed of Sound in Sea Water", JASA, Vol. 32, No. 10, October 1960, page 1357), there is a shorter and simpler formula due to

Leroy ("Development of Simple Equations for Accurate and More Realistic Calculations of the Speed of Sound in Sea Water," NATO TR No. 128 Salant ASW Research Centre, LaSpezia, Italy, November 1968). In this shorter formula there is no need to carry out a separate calculation for underwater pressure as is necessary when using Wilson's (Leroy: "Formulas for the Calculation of Underwater Pressure in Acoustics," JASA Vol. 44, No. 2, August 1968, page 65).

Both Wilson's and Leroy's formulas have been used to make some velocity comparison studies from typical temperature and salinity profiles. The results seem to indicate that the difference in velocity is quite small, being of the order of one-tenth of one per cent at large depth. In the ray tracing program, a choice can be made of either Wilson's or Leroy's.

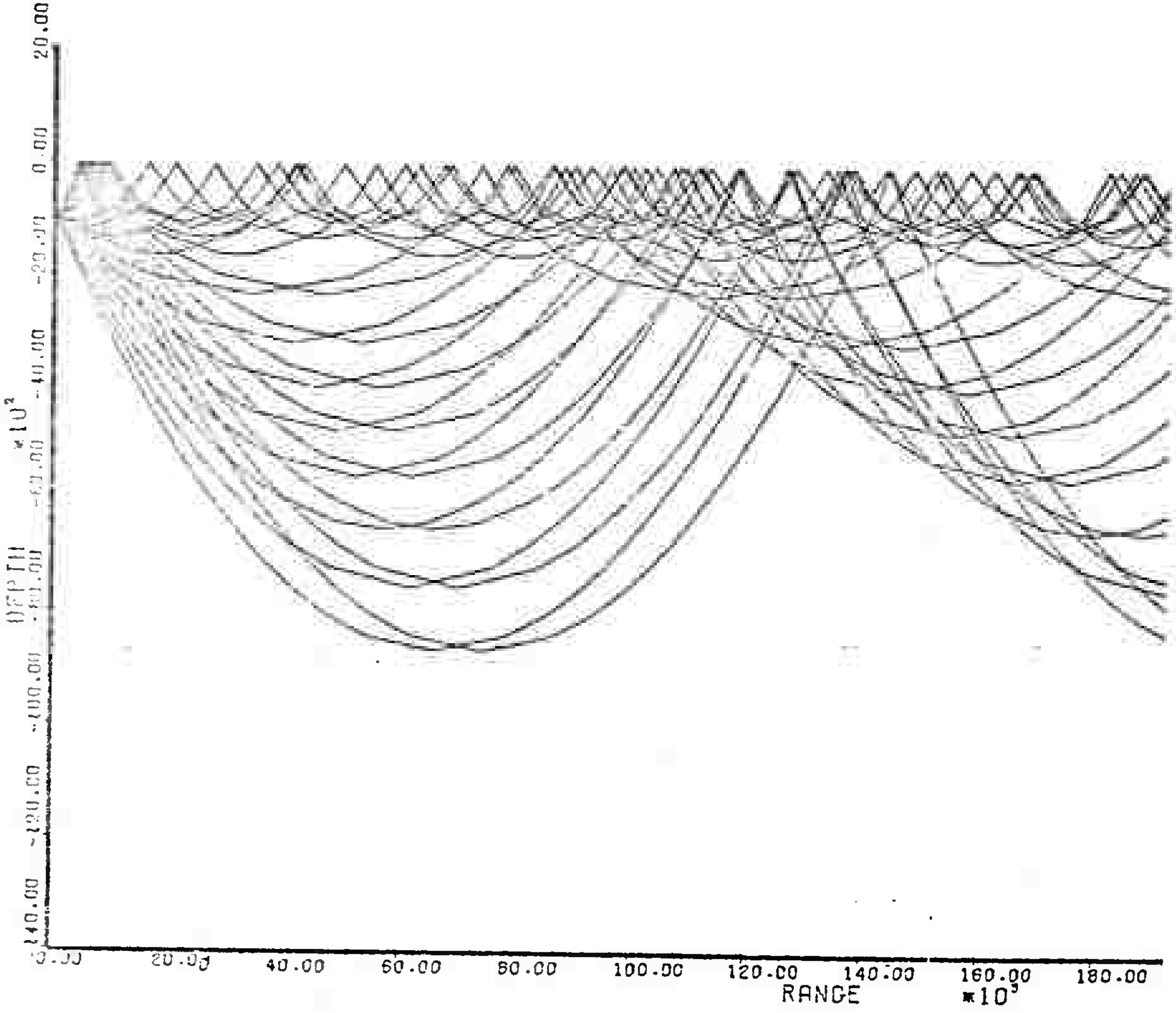
Arctic Topography

Approximately 5000 data cells in the program are used to describe the depth of the Arctic Ocean bottom. This covers the rectangular area: the midpoints of the four sides have the following latitudes and longitudes: 70°N 0°E, 75°N 90°E, 70°N 180°E, and 75°N 90°W. Each data cell represents the average depth of the ocean bottom for an area of approximately 35 miles square.

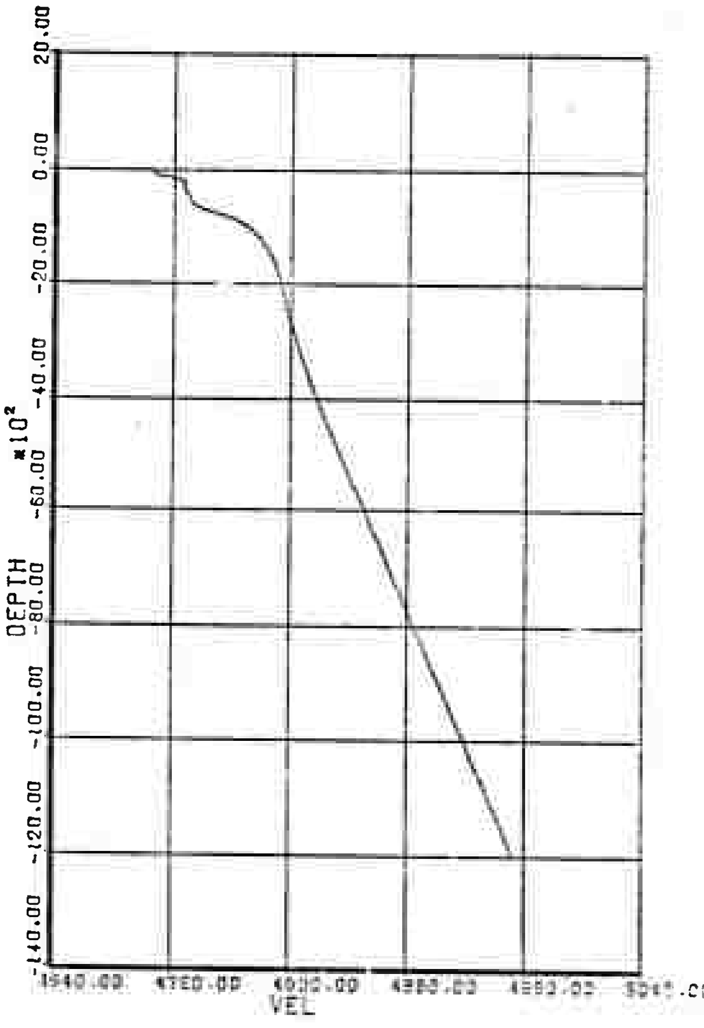
The data are based on the Arctic Bathymetry map as compiled by the Canadian Hydrographic Service, 1966-1967 (72°N 0°-90°W and 72°N 90°-180°W) and the North Polar Chart compiled by the Hydrographic Department of the British Navy, 1969.

A barthometic computer plot showing the approximate depth of the Arctic Ocean bottom is included in this report.

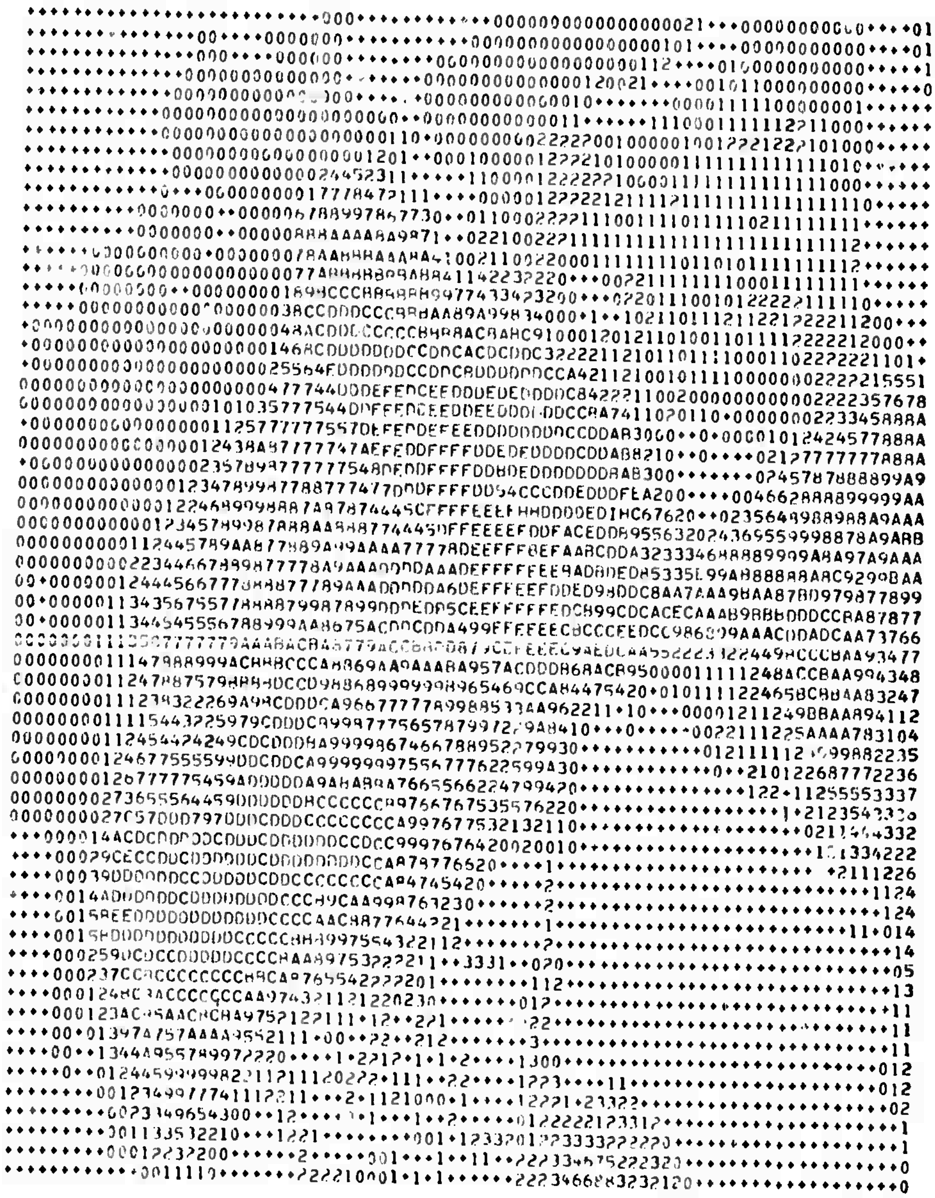
RAY DIAGRAM (BASE CAMP DATA)



VELOCITY PROFILE



ARCTIC BATHOMETRIC PLOT



+ INDICATES LAND. ALL NUMERALS AND LETTERS ARE DEPTHS 0= 0FT- 500FT

1= 501FT- 1000FT	2= 1001FT- 2000FT	3= 2001FT- 3000FT
4= 3001FT- 4000FT	5= 4001FT- 5000FT	6= 5001FT- 6000FT
7= 6001FT- 7000FT	8= 7001FT- 8000FT	9= 8001FT- 9000FT
A= 9001FT-10000FT	B=10001FT-11000FT	C=11001FT-12000FT
D=12001FT-13000FT	E=13001FT-14000FT	F=14001FT-15000FT
G=15001FT-16000FT	H=16001FT-17000FT	I=17001FT-18000FT

2. RELATED THEORETICAL DEVELOPMENT

Corrections to Ray Acoustics

Let (x_1, y_1, z_1) be rectangular Cartesian coordinates and t be the time. Let ℓ be some characteristic length (e.g., water depth), and define the non-dimensional coordinates by $x = x_1/\ell$, etc. Consider the wave equation

$$\Delta\phi = \frac{\ell^2}{c^2} \phi_{tt} \qquad (2.1)$$

where ϕ is the velocity potential, $c = c(x,y,z)$, a function of the space coordinates. Let c_o be some constant which can be taken approximately as the average value of c over the region of interest.

For periodic motion we assume

$$\phi = \mathrm{Re}\{u\ e^{iwt}\} \qquad (2.2)$$

where w is the (constant) angular frequency; $u(x,y,z)$ is the complex amplitude. Substitution of (2.2) into (2.1) shows that u satisfies the equation

$$\Delta u + k^2 n^2 u = 0 \qquad (2.3)$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad k = \frac{\ell w}{c_o} \quad \text{and} \quad n = \frac{c_o}{c}$$

The quantity k is non-dimensional, and in practice k^2 is usually >>1. For example, at a frequency of 20 Hz and with a depth of 2000 feet, k^2 is approximately 2500.

In acoustic propagation problems one is often dealing with a sequence of progressing waves of approximately constant phase; for example, plane waves, $u = e^{ikx}$, and spherical waves, $u = e^{-ikr}/r$. For such problems a natural substitution is

$$u = \alpha(x, y, z)e^{ik\beta(x,y,z)} \qquad (2.4)$$

where the function $\beta(x,y,z)$ is to provide the travelling wave character. Substitution of (2.4) into (2.3) gives

$$k^2[n^2-(\beta_x{}^2+\beta_y{}^2+\beta_z{}^2)]\alpha + ik[2(\alpha_x\beta_x+\alpha_y\beta_y+\alpha_z\beta_z) + \alpha\Delta\beta] + \Delta\alpha = 0 \tag{2.5}$$

For large k the first term is dominant so that as a first approximation

$$\beta_x{}^2 + \beta_y{}^2 + \beta_z{}^2 = n^2 \tag{2.6}$$

This is the eikonal equation, and its characteristics are the acoustic rays. If s is the distance along a characteristic, the differential equations of a characteristic become

$$\frac{dx}{ds} = \frac{\beta_x}{n}, \quad \frac{dy}{ds} = \frac{\beta_y}{n}, \quad \frac{dz}{ds} = \frac{\beta_z}{n}$$

$$\frac{d\beta_x}{ds} = n_x, \quad \frac{d\beta_y}{ds} = n_y, \quad \frac{d\beta_z}{ds} = n_z$$

$$\frac{d\beta}{ds} = n \tag{2.7}$$

from which the conventional ray equation follows:

$$\frac{d^2x}{ds^2} = -\frac{1}{n}\frac{dx}{ds}\left(n_x\frac{dx}{ds} + n_y\frac{dy}{ds} + n_z\frac{dz}{ds}\right) + \frac{1}{n}n_x \tag{2.8}$$

The term that is next important in (2.5) can also be put to zero to get a second approximation by choosing α such that

$$2(\alpha_x\beta_x + \alpha_y\beta_y + \alpha_z\beta_z) + \alpha\Delta\beta = 0 \tag{2.9}$$

From (2.7) and (2.9) it follows along a ray

$$2n\frac{d\alpha}{ds} + \alpha\Delta\beta = 0 \qquad (2.10)$$

i.e., $$\mathrm{div}(\alpha^2 \text{ grad } \beta) = 0 \qquad (2.11)$$

and by the divergence theorem for a closed surface S

$$\int_S \alpha^2 \text{ grad } \beta \cdot dS = 0 \qquad (2.12)$$

Consider a section of a ray tube and make use of (2.12). The contribution to the integral from the tube sides is zero since grad β is in the direction of the ray; so that if the two end sections of the ray tube be given the indices 1 and 2, we have

$$\int_{S_1} \alpha^2 n dS = \int_{S_2} \alpha^2 n dS \qquad (2.13)$$

Since the rate of the energy propagation per unit area is proportional to $(\text{amplitude})^2/c$, (2.13) implies that the rate of energy propagation is constant along a ray tube.

This leads to the conclusion that the use of ray theory is equivalent to the omission of the last term in (2.5). One way of examining the adequacy of the ray theory is to solve the problem by ray theory to obtain α and β at each point and to examine the magnitude of $\Delta\alpha$. Another approach is to iterate, compute $\Delta\alpha = f$, say, and recompute α by

$$2n\frac{d\alpha}{dS} + \alpha\Delta\beta + \frac{1}{ik} f = 0 \qquad (2.14)$$

A comparison of the revised values of α should provide a criterion for the adequacy of the ray approximation.

Such iterations can be carried out more systematically by using the asymptotic expansion below.

$$\alpha = \alpha^{(0)} + \frac{1}{ik}\alpha^{(1)} + \left(\frac{1}{ik}\right) \alpha^{(2)} + \ldots \qquad (2.15)$$

Substitution into (2.5) with β still satisfying (2.6) yields the sequence of equations:

$$2n \frac{d\alpha^{(0)}}{ds} + \alpha^{(0)}\Delta\beta = 0$$

$$2n \frac{d\alpha^{(1)}}{ds} + \alpha^{(1)}\Delta\beta + \Delta\alpha^{(0)} = 0$$

$$2n \frac{d\alpha^{(2)}}{ds} + \alpha^{(2)}\Delta\beta + \Delta\alpha^{(1)} = 0 \qquad (2.17)$$

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so that $\alpha^{(0)}$, $\alpha^{(1)}$, . . ., may be determined in succession. Note that the ray paths are unaffected by this iterative procedure.

3. SUBCONTRACT WORK TO LAMONT-DOHERTY

The principal effort at Lamont-Doherty has been to develop a rapid, accurate method of computing propagation loss as a function of range in the ice covered Arctic Ocean. Important input parameters to the propagation model are ice roughness, bottom topography, and the velocity structure as a function of depth in the ice, ater, and bottom sediments. Computational speed is of the utmost importance to evaluate variations of these parameters on propagation loss as a function of range since a number of models must be considered.

Such a rapid computational method is the Fast Field Program (FFP) technique discovered by Marsh (1967). The FFP was implemented for computation on a digital computer by Di Napoli (1971) for propagation in an all liquid wave guide. In the Arctic Ocean solid layers as well as liquid layers must be considered, and a convenient starting point for the FFP is the integral solution derived by matrix methods by Kutschale (1970, 1972) for propagation from point harmonic sources in a multilayered liquid-solid half space.

In the FFP technique the integral solution is evaluated rapidly as a function of range by numerical integration employing the Fast Fourier Transform (FFT). Singularities in the integrand corresponding to the normal mode poles are removed from the axis of integration by including an attenuation factor in each layer. The attenuation in each layer can represent the effects on propagation loss as a function of range of absorption of sound in the water, attenuation by the rough ice boundaries, and attenuation of sound by the bottom.

Two computer programs were written in Fortran IV to evaluate the liquid- and solid-bottom integral solutions derived by Kutschale (1970, 1972). A comparison of the FFP computations with those computations by normal mode theory from the corresponding integral

solutions are nearly identical, but the FFP technique is far more convenient and at least an order of magnitude faster since the computations are done directly from the integral solution without first computing the roots of the frequency (period) equation and then summing the normal modes.

Computations done to date by the FFP are in close agreement with measurements of propagation loss in the central Arctic Ocean. Present effort is a further comparison of propagation loss computations against experimental data and a comparison between FFP and computations by ray, normal mode, and the WKB mode theory. An evaluation is in progress of the effects on propagation loss as a function of range of thermal and salinity gradients, surface and bottom reflections, and bottom topography.

4. REFERENCES

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